(1) The three charges \((Q, 2Q, -3Q)\) as shown are located on the \(x\)- and \(y\)-axes at positions \((0, 0), (0, a),\) and \((a, 0)\), where \(a\) is a constant length, and \(Q\) is a positive constant. Treat the charges as point charges.

\[ E = E_1 + E_2 + E_3 \]

\[ E_x = E_1 \cos 45^\circ + E_2 \]
\[ = \frac{kQ}{(a\sqrt{2})^2} \cdot \frac{1}{\sqrt{2}} + \frac{k(2Q)}{a^2} = \frac{kQ}{2a^2} \cdot \left(1 + \frac{2}{\sqrt{2}}\right) \]
\[ E_y = E_1 \sin 45^\circ - E_3 \]
\[ = \frac{kQ}{(a\sqrt{2})^2} \cdot \frac{1}{\sqrt{2}} - \frac{k(3Q)}{a^2} = -\frac{kQ}{a^2} \left(3 - \frac{1}{2\sqrt{2}}\right) \]

(b) Find the electric potential at the point \(P\). As a reference, assume zero potential at infinity.

\[ V = \sum \frac{kQ_i}{r_{ij}} = \frac{kQ}{a} + \frac{kQ}{a\sqrt{2}} - \frac{k(3Q)}{a} \]
\[ = \frac{kQ}{a} \left(-1 + \frac{1}{\sqrt{2}}\right) = -\frac{kQ}{\sqrt{2}a} (\sqrt{2} - 1) \]

(c) If \(Q\) is an amount of charge equal to \(9.0 \mu C\), and \(a\) is \(15.0 \text{ cm}\), find the total energy that would be required to separate these charges to positions infinitely far from each other.

\[ |U| = \sum \frac{Q_i \cdot Q_j}{r_{ij}} \cdot \frac{k}{a} \quad \text{has 3 terms:} \]
\[ U = \frac{k \cdot 2Q^2}{a} - \frac{k \cdot 3Q^2}{a\sqrt{2}} - \frac{k \cdot 6Q^2}{a \cdot \sqrt{2} \cdot \sqrt{2}} = -\frac{kQ^2}{a} \left(1 + \frac{6}{\sqrt{2}}\right) \]

hence energy req'd = \((-U)\) = \(\frac{(9.0 \times 10^{-9} \text{ C})(9 \times 10^{-6} \text{ C})}{0.15 \text{ m}} (1 + \frac{6}{\sqrt{2}}) \)

\[ = 25 J \]
(2) The sketch illustrates a finite-length charged wire situated on the x axis. This wire (shown by the heavy line) extends from position a to 4a along the x axis, where a is a constant length. The wire is uniformly charged, with charge per unit length (-$\lambda$), where the charge is negative since $\lambda$ is a positive quantity.

(a) Find the electric potential at the origin due to this charge distribution, relative to the potential infinitely far away.

\[
\text{d}Q \text{ gives } \text{d}V = -k\frac{Q}{x} \\
\text{so } V = -k\lambda \int_a^{4a} \frac{dx}{x} = -k\lambda \ln \frac{4a}{a} = -k\lambda \ln 4
\]

(b) If a large insulating sphere were to completely surround this wire, what would be the total flux through the sphere, in terms of $\lambda$ and a?

use $\Phi = \frac{Q_{\text{in}}}{\varepsilon_0}$, and $Q_{\text{in}} = (-\lambda)(3a)$

so $\Phi = -\frac{3a\lambda}{\varepsilon_0}$
(3) Two infinitely-long crossed wires are situated as shown, so that they touch at the origin of the coordinate system, crossing at right angles to each other. The vertical wire has charge $+4\lambda$ per unit length, where $\lambda$ is a positive constant, while the horizontal wire has $-\lambda$ per unit length.

(a) First, consider the case where the vertical wire is alone. Find the electric field due to this single long wire, at a radius $r$ away from the wire. You must show your method in obtaining the answer, starting with a basic formula for our course (one from the formula sheet!).

Gauss' law $\Rightarrow \vec{E} = \frac{Q_{in}}{\varepsilon_0}$. But $\Phi = \int \vec{E} \cdot d\vec{A}$ $= EA$ here (symmetry)

where $A = 2\pi rl$

$E = \frac{4\lambda l}{2\pi\varepsilon_0 r} \Rightarrow E = \frac{2\lambda}{\pi\varepsilon_0 r}$ (r-direction)

(b) Now consider the case where both wires are present. Find the electric field at point P, where P has coordinates (1.0 cm, 3.0 cm), and where the origin of the coordinates is the crossing point of the wires. In this case assume that $\lambda$ is equal to $8.5 \times 10^{-9}$ C/m. Give magnitude and direction.

$4\lambda$ wire $\Rightarrow E_1 = \frac{2\lambda}{\pi\varepsilon_0 1cm} \text{ is horizontal}$

$-\lambda$ wire $\Rightarrow E_2 = \frac{-\lambda}{2\pi\varepsilon_0 \frac{1}{2}(3cm)} \text{ vertical (by same method as above)}$

The vectors are plotted above; $\Theta$ is angle below $x$.

$90^\circ - \Theta = \tan^{-1} \left( \frac{1E_1}{1E_2} \right) = \tan^{-1} \left( \frac{2\lambda/\pi\varepsilon_0 \frac{1}{2}(3cm)}{2\lambda/\pi\varepsilon_0 \frac{1}{2}(5cm)} \right) = \tan^{-1}(12)$

$\Theta = 9.2^\circ$ \hspace{1cm} $\Rightarrow |E| = \sqrt{E_1^2 + E_2^2}$

and $1E_1 = \frac{2\lambda}{0.01m} = 61000\frac{V}{m}$ \hspace{1cm} $1E_2 = 5100\frac{V}{m}$ \hspace{1cm} $\Rightarrow |E| = \sqrt{61000^2 + 5100^2}$

$\approx 61400\frac{V}{m}$
(4) The figure illustrates two very thin concentric metal spheres, radius $R_a$ and $R_b$. There is a charge of $+3Q$ on the outer sphere.

(a) If there is no electric field in the region outside the spheres, what if any charge must there be on the inner sphere?

Gauss's law tells us $Q_{in} = 0$, so inner sphere balances charge of outer, thus $Q = (-3Q)$ at $R_a$.

(b) What is the surface charge density on the outer sphere?

$$(+3Q) \text{ distributed over area } A = 4\pi R_b^2$$

Gives:

$$\sigma = \frac{Q}{A} \Rightarrow \frac{+3Q}{4\pi R_b^2}$$

(c) Find the voltage difference between the two spheres, if $Q = 7.5 \times 10^{-9}$ C, $R_b = 0.070$ m, and $R_a = 0.030$ m.

Region $R_a$ to $R_b$ has point-charge-type $E \in V$ fields, for an effective point charge of $(-3Q)$.

$\Rightarrow E = \frac{-k(3Q)}{r^2}$ hence, and so $V = \frac{-3kQ}{r} + \text{constant}$

Specifically, if gnd. at $R_b$, $V(r) = \int_{R_b}^{r} \frac{3kQ}{r^2} dr = \frac{-3kQ}{r} + \frac{3kQ}{R_b}$

so the constant $= 3kQ/R_b$. It's not really needed, though

Then, $|\Delta V| = \frac{3kQ}{r} \bigg|_{R_b}^{R_a} = \frac{3kQ}{R_a} - \frac{3kQ}{R_b}$

$= 6.750V - 2.890V \approx 3.890V$
(5) The $dx \times dx \times dx$ cube shown is insulating, and uncharged. It sits with its base touching the $x-y$ plane. There is an $E$-field pointing in the $z$-direction, shown by the dashed lines, except that $E$ changes with $z$ as follows: $\vec{E} = E_0 \left(1 + 4 \frac{z}{d}\right) \hat{k}$, where $E_0$ is a constant.

(a) Find the flux through each of the six faces of the cube: the bottom, the top, and the four side faces.

- 4 side faces have $\Phi = 0$ since $\Theta = 90^\circ$, $\cos \Theta = 0$ in $E \cos \Theta$
- Bottom, $\vec{A}$ is downward
  \[\Phi = -EA \text{ for } E \times z = 0,\]
  \[E(0) = E_0 \text{ so } \Phi = -E_0 A\]
  or $-E_0 d^2$
- Top, $E = 5E_0$ at $z = d$,
  \[so \; \Phi = +5E_0 d^2\]

(b) How much charge must be contained inside the cube, in terms of $d, E_0$, and fundamental constant(s)?

Net flux is sum of the terms above,

\[\Phi_{net} = 5E_0 d^2 - E_0 d^2 = 4E_0 d^2\]

but $\Phi_{net} = \frac{Q_{in}}{\varepsilon_0}$ by Gauss' law,

\[so \; Q_{in} = +\varepsilon_0 \cdot 4E_0 d^2\]