Notes for today

- Reading: continuing Ch. 3.

- Note no exam this week. (Originally 3 midterm exams in Howdy schedule). We have a midterm Oct. 29. More details on this later in the term.

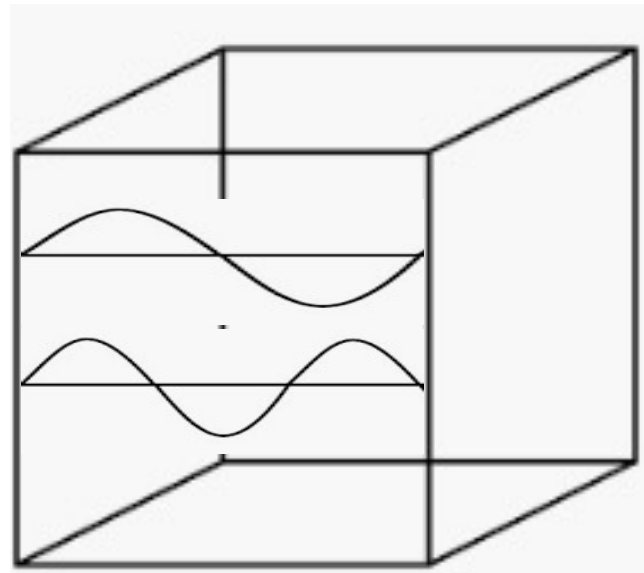
- Looking for HW presentations #4, 6.

- Reminder about lecture recording, I can send a recording link if you have to miss.

Blackbody radiation:

Results & assumptions I quoted last time:

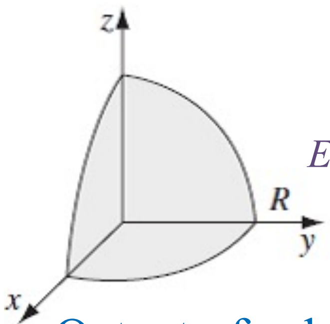
- each 3D standing wave corresponds to 2 solutions (similar to 2 polarizations for free traveling waves).
- Wave solutions indexed by n_x, n_y, n_z, with $\omega = |k|c$, $|k| = (\sum[n_i\pi/L]^2)^{1/2}$.
- Amplitude of each mode is quantized, contributes $U = \left(n + \frac{1}{2}\right)\hbar\omega$ to internal energy.
- Average amplitude per mode, in *thermal equilibrium*, same as result we derived for simple harmonic oscillators, $\frac{\hbar\omega}{e^{\beta\hbar\omega}-1} \equiv \hbar\omega\langle n\rangle$.



Note $\beta \equiv 1/kT$

State counting:

- Start with cavity modes in a box with perfectly conducting sides, dimensions L.



$$E_x \propto cos\left(n_x \frac{\pi}{L} x\right) sin\left(n_y \frac{\pi}{L} y\right) sin\left(n_z \frac{\pi}{L} z\right) \text{ etc.}$$

E-field

Cavity mode Counting: one TM + one TE per k-vector

$$\Delta k = \frac{\pi}{L} \text{ -> } V_k = \left(\frac{\pi}{L}\right)^3$$

Octant of sphere;
but with 8× state density.
(3D sphere radius will go to infinity)

Corresponds to
"Phase space volume" $h^3/8$

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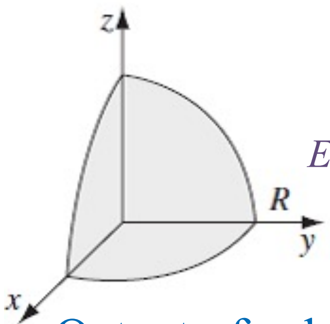

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Consider continuum limit (large cavity, very small Δk)
Also recall $\omega = kc$

$$U = \sum_{all\ modes} \frac{\hbar\omega_i}{(e^{\beta\hbar\omega_i} - 1)} \Rightarrow 2\int_0^\infty \underbrace{\frac{\pi}{2}\frac{Vk^2dk}{\pi^3}}\frac{\hbar kc}{(e^{\beta\hbar kc} - 1)}$$

\# modes in thin shell, thickness $dk = d\omega/c$

Note $\beta \equiv 1/kT$

Results:

Continuum limit in cavity,

$$U = \sum_{all\ modes} \frac{\hbar\omega_i}{(e^{\beta\hbar\omega_i} - 1)} \Longrightarrow 2\int_0^\infty \frac{\pi}{2}\frac{Vk^2dk}{\pi^3}\frac{\hbar kc}{(e^{\beta\hbar kc} - 1)}$$

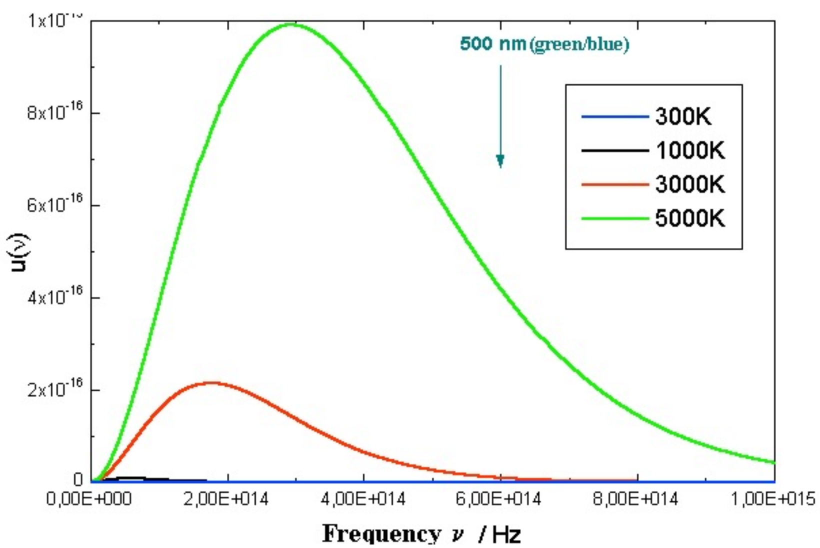
$$U = \frac{V\pi^2(kT)^4}{15(\hbar c)^3}$$

← $\frac{U}{V}$ independent of cavity details, recovers thermodynamic result showed last time.

Infinite number of "oscillators" but finite result due to exponential.
result closely related to Stefan-Boltzmann intensity law.

Peak $\hbar\omega \cong 2.8kT$

$u(\nu)$



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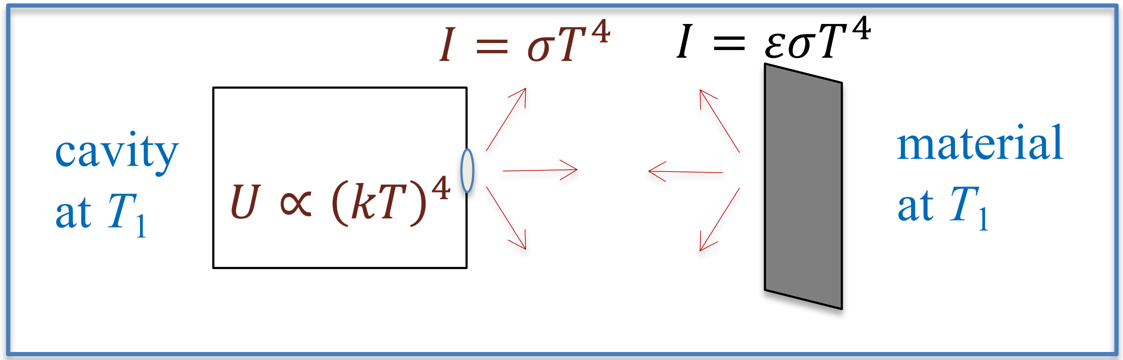

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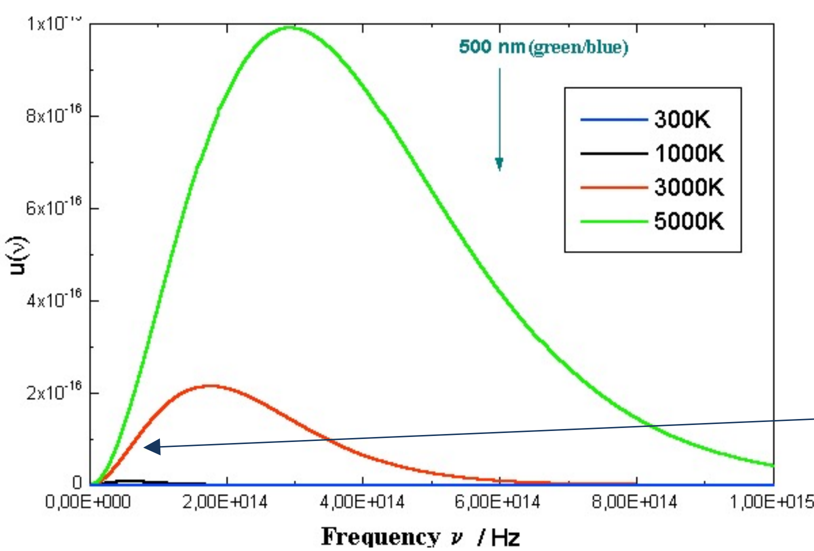

$$U = \int_0^\infty \frac{\hbar V\omega^3 d\omega}{c^3\pi^2}\frac{1}{(e^{\beta\hbar kc} - 1)}$$

Integrand proportional to <u>intensity spectrum</u>

Classical limit OK here; "ultraviolet catastrophe" averted by energy quantization: infinite number of oscillators $\neq$ infinite U

Results:

Continuum limit in cavity,

$$U = \sum_{all\ modes} \frac{\hbar\omega_i}{(e^{\beta\hbar\omega_i} - 1)} \Rightarrow 2\int_0^\infty \frac{\pi}{2}\frac{Vk^2 dk}{\pi^3}\frac{\hbar kc}{(e^{\beta\hbar kc} - 1)}$$

$$U = \frac{V\pi^2(kT)^4}{15(\hbar c)^3}$$

Note about superposition, this is mixed state not a coherent QM superposition.

Assumes *rapid interchange* with wall at temperature *T*, or *long-time average* to allow for overall thermal equilibrium.

Thus we aren't under the same set of assumptions used for "microcanonical", e.g. constant *U, N, V*. Here, constant *T, V*. Indistinguishable results in the thermodynamic limit.

(*N* isn't fixed, though we *can* determine statistical *photon number*.)

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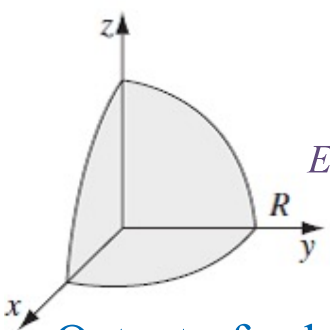

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$$E(x) = E(x+L) \ etc. \text{ -> } \qquad E = e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$, complete k sphere

$$\vec{k} = \left(n_x \frac{2\pi}{L} x,\ n_y \frac{2\pi}{L} y,\ n_z \frac{2\pi}{L} z\right)$$

Traveling waves
"Photons" have two "spin" or helicity states, ±1

Phase space volume h^3 , counting yields same result as above since include entire sphere in momentum space

Counting "Photons":

Result for internal energy,

$$U = \sum_{all\ modes} \frac{\hbar\omega_i}{(e^{\beta\hbar\omega_i} - 1)} \Longrightarrow 2\int_0^\infty \frac{\pi}{2}\frac{Vk^2dk}{\pi^3}\frac{\hbar kc}{(e^{\beta\hbar kc} - 1)}$$

$$U = \frac{V\pi^2(kT)^4}{15(\hbar c)^3}$$

$$\Longrightarrow C_V = \frac{4U}{T} \sim T^3$$

"specific heat of free space"

$$\langle N\rangle = \sum_{all\ modes} \frac{1}{(e^{\beta\varepsilon_i} - 1)} = \frac{2\zeta(3)V}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3$$

Riemann zeta
$\zeta(3) \approx 1.2$

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Result $U \cong 2.7\langle N\rangle kT$

- $\langle N\rangle$ counts number of quanta in a mixed state; could *detect* them as photons with a photodetector.
- Entropy: can also obtain result shown last time.

Blackbody radiation, thermodynamic solution

- Experimental quantities:

$$U = bVT^4$$
$$P = U/(3V)$$

$I = \sigma T^4$ *Stefan-Boltzmann intensity relation*

- Then can easily solve for $S = \frac{4}{3} b^{1/4} U^{3/4} V^{1/4}$, using methods we have seen.
- Also note, $S = \frac{4U}{3T}$ simpler form.
- Note N is formally zero (or can treat N as number of photons; $\mu = 0$ since U independent of N).

$S \cong 3.6\langle N \rangle k_B$
"independent of T" form

also note, $PV \approx NkT$

Thermal photons:

$$U = \frac{V\pi^2(kT)^4}{15(\hbar c)^3} \cong 2.7NkT \qquad S = \frac{4U}{3T} \cong 3.6\langle N\rangle k_B$$

- Yields *R*-dependence of *T* for expanding universe background radiation
- & can see, $\langle N\rangle$ = constant, *U* decreases as universe expands (adiabatic expansion).

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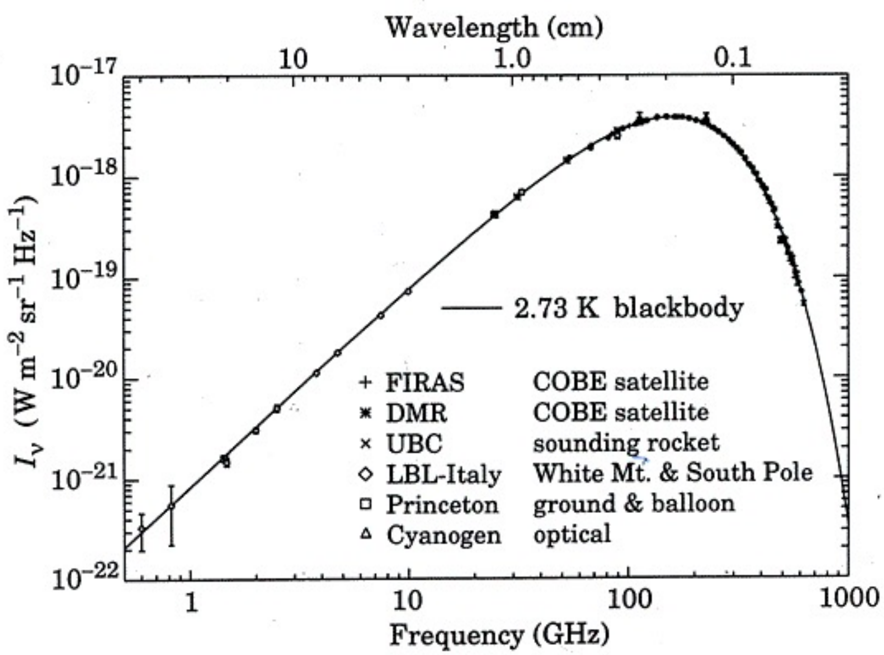

2.7 K thermal radiation
Entropy: much larger than that of visible matter (stars, dust)
(But black holes likely contain even larger total entropy.)

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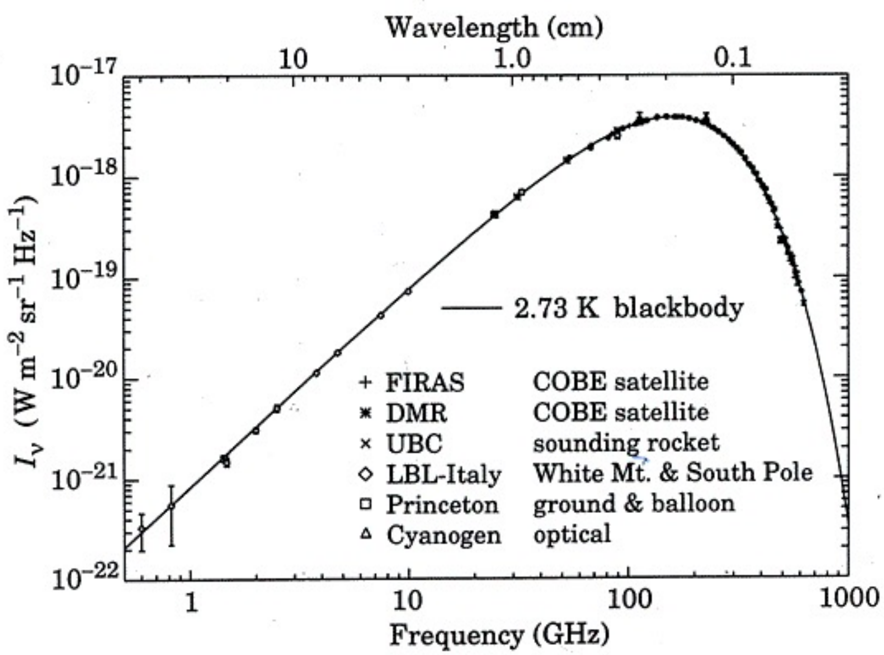
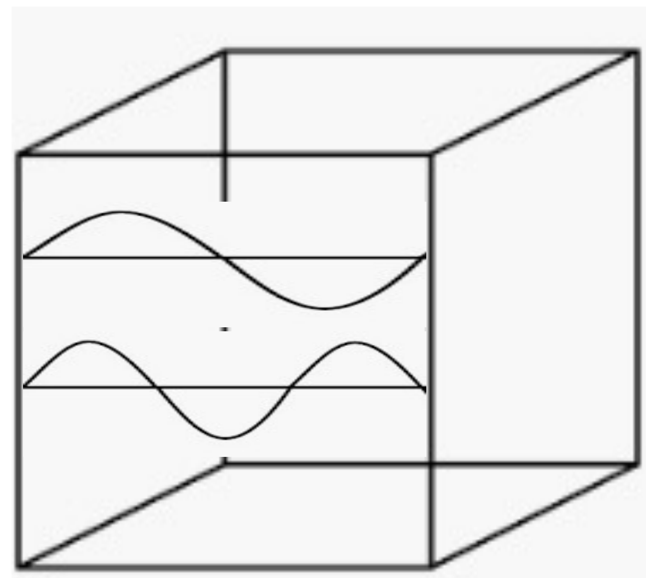


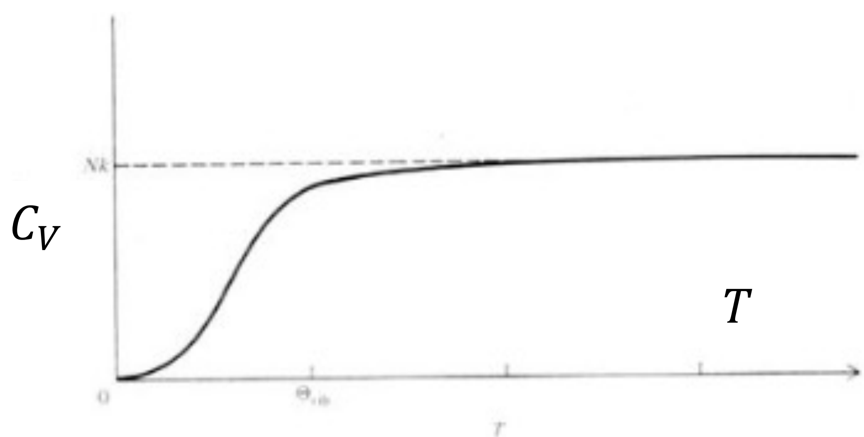
Current picture: radiation decoupled from plasma when "fireball" cooled to ~3000 K (thermal ionization ceased)

Photons are still here but no longer in equilibrium with anything.

Reminder again, assumptions used:

- Two solutions for each n_x, n_y, n_z, with $\omega \ = |k|c = (\sum n_i\, \pi/L)^{1/2} c$.
- Amplitudes quantized, giving eigenstates with $U = \left(n + \frac{1}{2}\right) \hbar\omega$.
- In <u>*thermal equilibrium*</u>, *n* expectation value same as derived for simple harmonic oscillators, $\frac{\hbar\omega}{e^{\beta\hbar\omega} - 1} \equiv \hbar\omega\langle n\rangle$.
- Counting of modes, not *quanta*, this case.



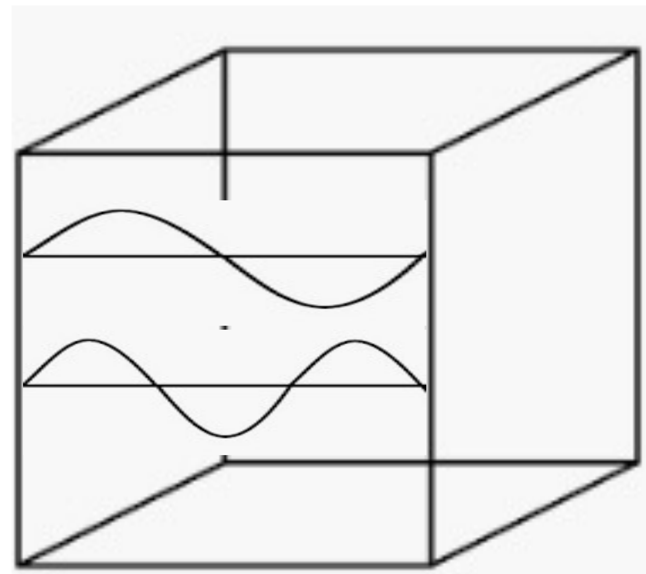


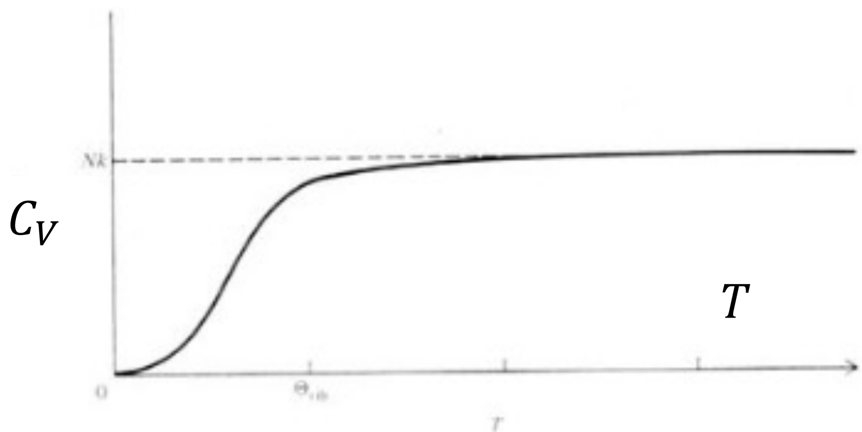
$$U = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}$$

single oscillator

Comparing, EM radiation vs idealized classical solid (& gas):

- EM solution $\omega = |k|c = (\sum n_i \pi/L)^{1/2}c$ unbounded in number of modes, so e.g. C_V also unbounded.
- Solid can *also* have *k*-dependent ω (Debye model), but # oscillators still fixed = 3*N*.
- All 3 cases, *S* vs. *U* negative curvature as expected (thermal stability).





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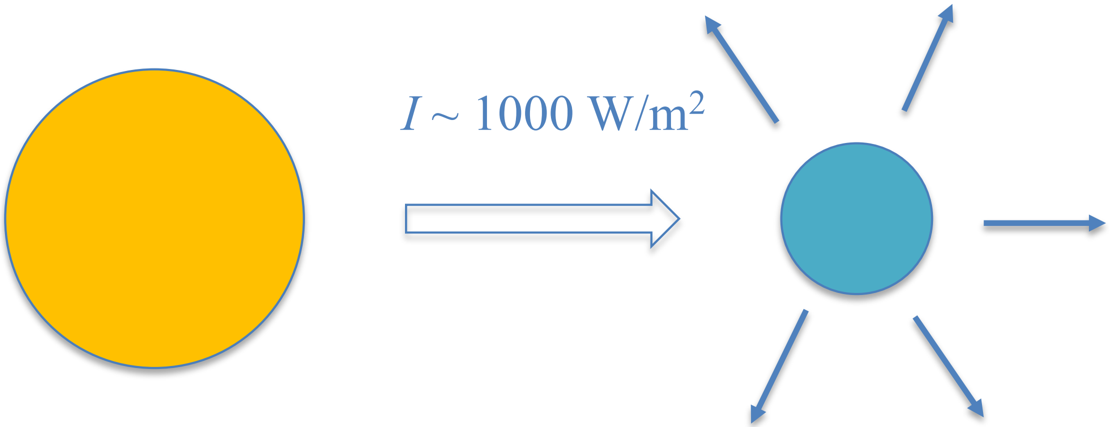
single oscillator

Remark on Sun-Earth system:



Sun ~ 5000 K

Earth ~ 255 K

Steady state: **same absorbed & radiated power**

Can see, entropy <u>*must*</u> increase in this process.

$$S = \frac{4U}{3T} \cong 3.6 N k_B$$

Earth emissivity ~0.8. [depends strongly on frequency; <u>greenhouse effect</u>.]