Notes:

Homework : Set #10 is posted, due next Tuesday.

Today: chapter 18, plus section 17.3. (I covered other parts of chapter 17 before. You should read all of the short chapter 17.)

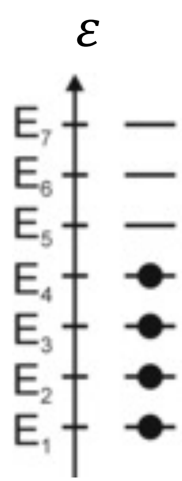
Fermions, Bosons:

- Fermions, no double-occupation of same state (all single-particle states have distinct quantum numbers). Pauli exclusion.

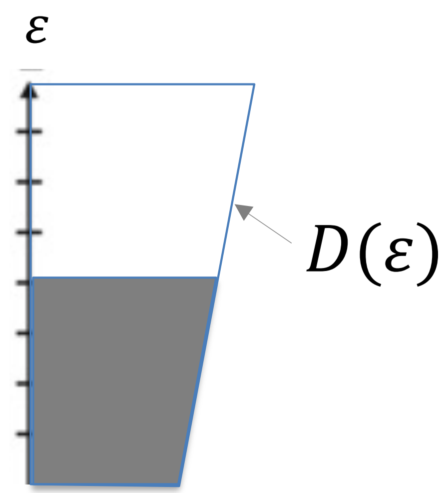
  electrons, positrons, neutrons, ^{3}He atoms, …
- Bosons, unlimited occupation of any state (Bose condensation for $T \to 0$).

  photons, phonons, gluons, Higgs bosons, ^{4}He atoms, …
- Fermions $J = 1/2, 3/2, 5/2…$;   Bosons $J = 0, 1, 2, ...$
- Comes from ±1 phase change upon interchanging two particles (Dirac).

Fermions, $T \approx 0$:



Discrete
energy levels



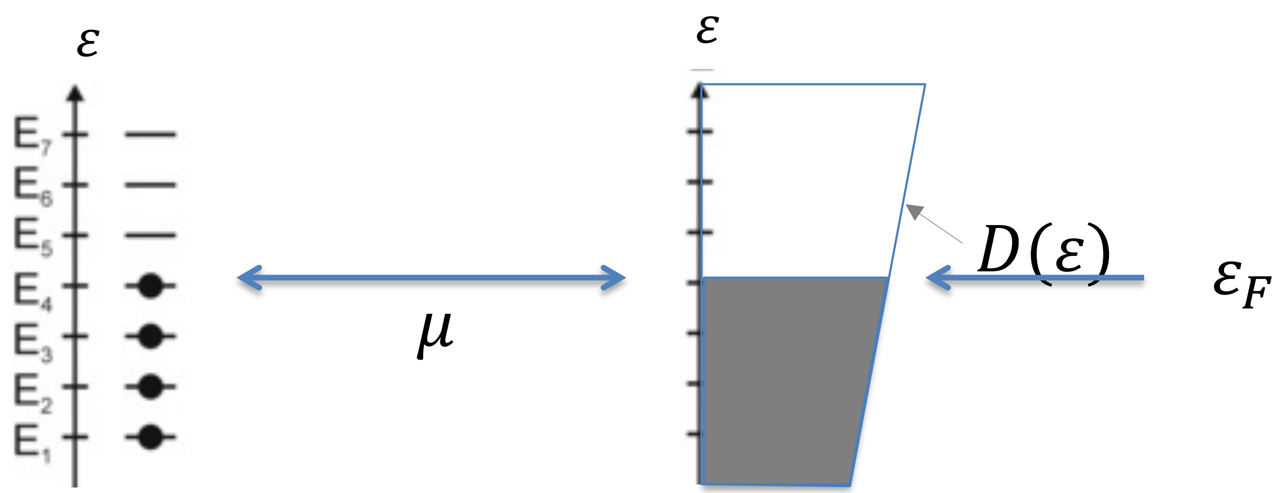
Continuum cavity states
(e.g. "Fermi gas")

(electrons with spin 1/2,
double each orbital state)

$$S = 0$$

$$F = E$$

Fermions, $T \approx 0$:



Discrete
energy levels

Continuum cavity states
(e.g. "Fermi gas")

(electrons with spin 1/2,
double each orbital state)

$$\left.\begin{aligned} S &= 0 \\ F &= E \end{aligned}\right\} \quad \mu = (highest\ occupied) \equiv \varepsilon_F$$

result specific for $T \sim 0$

ε_F = "Fermi energy"
(continuum case)

Recall classical indistinguishable case,

$$Z = \frac{1}{N!}[Z_i]^N \qquad Z_i = \int_0^\infty \frac{V}{h^3} 4\pi p^2 dp e^{-\beta p^2/2m} = \frac{Vm^3}{h^3} 4\pi \left(\frac{\sqrt{2\pi}}{(\beta m)^{3/2}}\right)$$

$$= 2V\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \equiv 2V/\lambda_{th}^3$$

Thermal DeBroglie wavelength

Quantum case:

$$Z = \sum_{\substack{states\ i \\ "orbitals"}} Exp[-E_i/kT]$$

$$S = k(\beta U + lnZ) = k\left(-\beta \frac{\partial}{\partial \beta} lnZ + lnZ\right)$$

$$S = Nk_B ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + \frac{5}{2}Nk_B = Nk_B ln\left[\frac{V}{N\lambda_{th}^3}\right] + \frac{5}{2}Nk_B$$

Gibbs factor takes into account indistinguishable particles

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Example 2 particles, 3 states:

A	B	
A		B
B	A	
	A	B
B		A
	B	A
AB		
	AB	
		AB

A	A	
	A	A
A		A
	?	

Fermi & Bose cases?

(Z contains 9 terms)

Classical: distinguishable $3^2 = 9$ possible microstates
vs. indistinguishable $3^2/N! =$ "4.5 microstates"
(in classical = "Maxwell-Boltzmann" limit, this works well.)

Ensembles:

Internal energy (or Entropy): $dU = TdS - PdV + \mu dN$

Closed system, well-defined energy (or e.g. $E \pm \Delta E/2$):
Microcanonical ensemble $S = k ln \Omega$ maximized

$U(S, V, N)$

Helmholtz free energy: $F = U - TS$.

$F(T, V, N)$ $dF = -SdT - PdV + \mu dN$

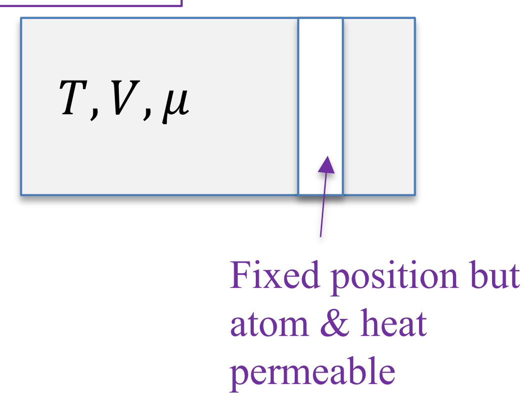
$F = -kT ln Z$ minimized

Canonical ensemble

Grand Potential $\Psi = U - TS - \mu N$.

Section 17.3

$\Psi(T, V, \mu)$ $\Psi = -kT ln Z_g$ minimized



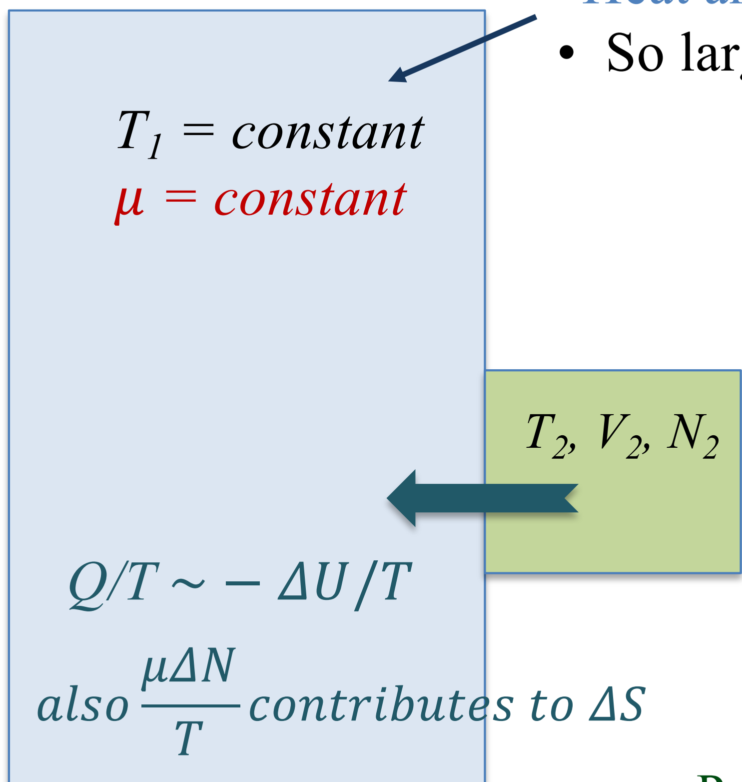
Grand canonical ensemble

handles Bose or Fermi statistics more easily

Grand Canonical Ensemble:

"Heat and atom bath"

- So large its T and μ doesn't change



For any 2 microstates in system 2:

$$\frac{P_A}{P_B} = e^{[S_R(A)-S_R(B)]/k_B} = e^{-\Delta U+\mu\Delta N/k_B T}$$

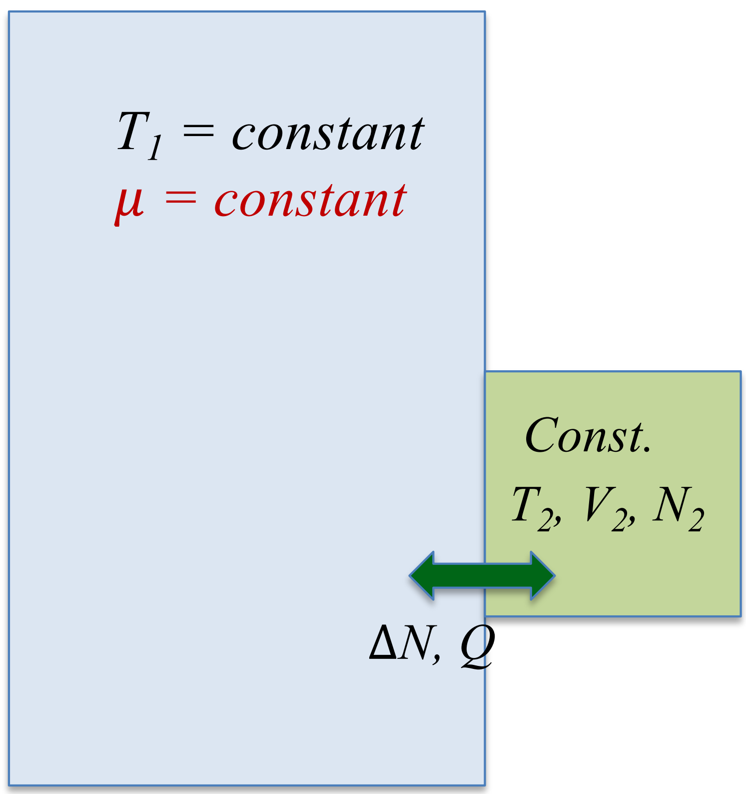
$$P_i = \frac{1}{Z_g} e^{(-E_i+\mu N)/kT}$$

Probability of finding system in state i in equilibrium

$$Z_g = \sum_{i,N} e^{(-E_i+\mu N)/kT}$$

For individual states i

Grand Canonical Ensemble (μ, T, V constant):



Probability of state *i* in equilibrium

$$P_i = \frac{1}{Z_g} e^{(-E_i + \mu N)/kT}$$

"Ƶ" Grand Canonical partition fn.

$$Z_g = \sum_{i,N} e^{(-E_i + \mu N)/kT}$$

Sum over allowed individual states *i*, also over all possible number *N* of particles in system

$$V / N\lambda_{th}^3 \lesssim 1$$

- Valid in classical or quantum ("Degenerate") limits.
- Note, $Z_g = \sum_N Z\left[e^{\beta\mu}\right]^N$; $e^{\beta\mu}$ = "fugacity"
- Can show, same limiting properties as canonical & microcanonical, in thermodynamic limit.

Grand Canonical Ensemble:

Probability of state i in equilibrium

$$P_i = \frac{1}{Z_g} e^{(-E_i + \mu N)/kT}$$

$$Z_g = \sum_{i,N} e^{(-E_i + \mu N)/kT}$$

$$Z_g = \sum_{i,N} e^{(-E_i + \mu_1 N_1 + \mu_2 N_2 + \cdots)/kT}$$

General case, different particle types

Particle number:

$$Z_g = \sum_{i,N} e^{(-E_i + \mu N)/kT} \Longrightarrow \langle N \rangle = kT \frac{\partial}{\partial \mu} ln(Z_g)$$

Can show, distribution is **narrow spike** in thermo. limit

$$\langle U \rangle = -\frac{\partial}{\partial \beta} ln(Z_g) + \mu_1 \langle N_1 \rangle + \mu_2 \langle N_2 \rangle + \ldots$$

Fermions:

$$Z_g = \prod_i \left(1 + e^{-(E_i - \mu)/kT}\right)$$

Fermi-Dirac product of single-particle states; easy to see, includes all possible occupation of each eigenstate (either 1 or 0).

Fermions:

$$Z_g = \prod_i \left(1 + e^{-(E_i - \mu)/kT}\right)$$

Fermi-Dirac product of single-particle states

$$n = \frac{1}{\left(1 + e^{(E-\mu)/kT}\right)}$$

Fermi-Dirac distribution = $\langle N \rangle$ for a single eigenstate.

Same as "Fermi function"; $f(E)$

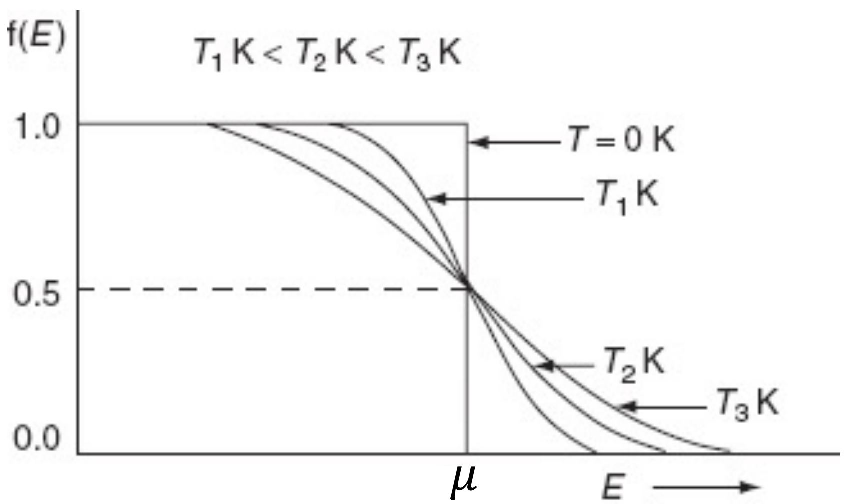
- Valid for non-interacting particles (similar to our product-state partition function analysis for canonical ensemble).
- Some methods exist to treat interacting cases; this is a central issue in many-body physics.

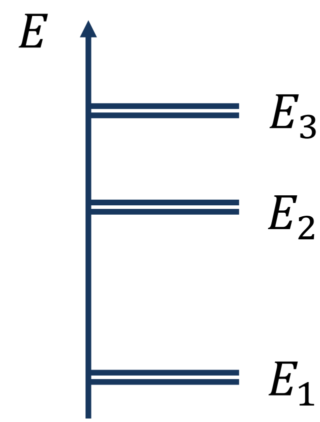
Fermions:

$$n = \frac{1}{(1 + e^{(E-\mu)/kT})}$$

Fermi-Dirac distribution = $\langle N \rangle$ for a single eigenstate.

Same as "Fermi function"; $f(E)$





Works in continuum or discrete limit.

4 particles, 6 states where is μ?

Bosons:

$$Z_g = \prod_i \sum_N e^{-(NE_i - N\mu)/kT}$$
$$\approx \prod_i 1/\left(1 - e^{-(E_i-\mu)/kT}\right)$$

Bose-Einstein product of single-particle states; Converges to lower form if $E_i > \mu$ (no condensation).

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Bosons:

$$Z_g = \prod_i \sum_N e^{-(NE_i - N\mu)/kT}$$
$$\approx \prod_i 1/\left(1 - e^{-(E_i-\mu)/kT}\right)$$

Bose-Einstein product of single-particle states; Converges to lower form if $E_i > \mu$ (no condensation).

$$n = \frac{1}{(e^{(E-\mu)/kT} - 1)}$$

Bose-Einstein distribution = $\langle N \rangle$ for a single eigenstate.