Physics 416 Problem Set 3 Due Monday, Sept. 26

(1) Consider a 2-dimensional crystal consisting of a triangular net, with 60° angles and equal near-neighbor distances a.

(i) Find the reciprocal lattice for this structure and sketch in a simple way how it registers with the direct-space lattice.

(ii) Carefully construct a sufficient number of perpendicular bisector planes (lines in this case), and indicate the first 5 Brillouin Zones. (Carefully means at least with a ruler.)

(iii) Show on a separate simple sketch how pieces of the 3rd zone can "fold" back into the first zone, and indicate which G vectors are used to translate some of the pieces.

(iv) Choose one of the points in the first Brillouin zone that is furthest from the origin. For this point, determine the wave-vector, and sketch (perhaps on your sketch for part (i)) the wave-fronts of a corresponding phonon mode in the direct-space lattice.

(2) Kittel 4.2

(3) Kittel 4.3.

(4) 2D rectangular monatomic lattice, transverse modes: Assume a simple rectangular lattice having identical atoms with spacing a in the x direction and b in the y direction. The lattice is aligned along the x-y plane, with nearest-neighbor interactions only. Assume displacements only in the z direction, and that the forces (in the z direction) are, $F_{(i,j),(i\pm1,j)} = -K_1(u_{(i,j)} - u_{(i\pm1,j)})$ for the "springs" along the x-direction, and $F_{(i,j),(i,j\pm1)} = -K_2(u_{(i,j)} - u_{(i,j\pm1)})$ for the y-direction "springs".

(a) Write down an expression for the total elastic energy of the lattice, and then by taking appropriate derivatives obtain the equation of motion for the transverse modes. Substitute a solution of the form, $u_o \exp i[\vec{k}\cdot\vec{r} - \omega t]$, and show that the solutions have the form, $M\omega^2 = 2K_1(1 - \cos k_x a) + 2K_2(1 - \cos k_y b)$.

(b) Show that the allowed solutions fill the first Brillouin zone, and determine the largest frequency among all the vibrational modes corresponding to this solution.