Physics 617 Problem Set 4 Due Monday, Feb. 20

(1) 2D rectangular lattice in tight binding: Consider a set of identical localized states $|m,n\rangle$ on a 2D simple rectangular lattice, where the integers m, n index the positions of these states on 2D lattice sites in x and y directions. Suppose that the Hamiltonian can be expressed as,

$$H = \sum_{m,n} \Big[\varepsilon_o |m,n\rangle\langle m,n| - t_1 \big(|m,n\rangle\langle m,n+1| + |m,n\rangle\langle m,n-1| \big) - t_2 \big(|m,n\rangle\langle m+1,n| + |m,n\rangle\langle m-1,n| \big) \Big],$$

where ε_o, t_1, and t_2 are constants. This Hamiltonian couples only the nearest neighbors, along y direction (t_1 term) and x (t_2 term). Assume lattice constants are a (along x) and b (along y).

(a) Write down a single-particle Bloch wave function composed of the localized states $|m,n\rangle$.

(b) Plug your wave-function into the Schrodinger equation, show that it is an eigenstate, and solve for the energy vs. k for the resulting band. Find the lowest and highest energies for the band, and identify the k-space coordinates of four saddle points in $\varepsilon(k)$. Sketch the positions of the extrema and saddle points in the first Brillouin zone for this lattice.