Physics 617 Problem Set 6 Due Wednesday, Mar. 29

1) 1D harmonic crystal with a single-atom basis: Taking the atom mass M and spring constant K, lattice-vibrations have the form of $\omega = 2\sqrt{\frac{K}{M}}\sin\left[ka/2\right]$, as shown in class.

(a) Find the 1D density of modes, $g(\omega)$, for this solution. Assume a crystal of length L.
(b) What is the speed of sound?
(c) Find the low-temperature specific heat for this lattice (leading T-dependent term). Assume the temperature is low enough that the 1D equivalent of the Debye approximation works well. You can use the following integral: $\int_0^\infty dx \frac{x}{e^x - 1} = \zeta(2) = \pi^2/6$

2) Average square displacement:

(a) Use the formula (L.14) from the text appendix to find the expectation value, $\langle u(R)^2 \rangle = \langle \{n_{ks}\} | u(R)^2 | \{n_{ks}\} \rangle$, of the squared ionic displacement operator, in a state of known phonon occupation numbers, $\{n_{ks}\}$. (The curly bracket refers to a specific set of the integers n_{ks}.) The eigenvalues of a_{ks} and its Hermitian conjugate are given by (L.8), except that n would be written n_{ks} in all cases.

(b) In the $T = 0$ case, use the Debye approximation for the density of modes, and evaluate the sum. As a result find a numerical value for the rms displacement at $T = 0$ for Aluminum, which has Θ_D = 430 K and M = 27 g/mol.

3) Ashcroft and Mermin problem #25.5.