Physics 617 Problem Set 8 Due Monday, May 1

1) **Classical Curie law:** For classical magnetic dipoles, where moments μ can take any angle, the field energy is $U = -\vec{\mu} \cdot \vec{B}$. For a density *n* of such moments, in a uniform field *B*, solve for the magnetization $M = \langle \mu \rangle / V$, by integrating over all solid angles, with the probability given by the classical Boltzmann distribution. The integrations are relatively straightforward and you should arrive at the Langevin function described on the handout. Show that the magnetization has paramagnetic sign, and that in the high temperature limit the result is $M = CH/T$, the classical Curie law. Find the classical Curie constant *C*, and show that it is the same as we derived for the case of spins *S*, in the limit of large *S*.

2) **Current density and diamagnetism:**

(a) Demonstrate the steps leading to equation (7) in the magnetism handout, showing how the probability current $\vec{j} = -\frac{i\hbar}{2m}\left[\psi^* \vec{\nabla} \psi - (\vec{\nabla}\psi^*)\psi\right] + \frac{e}{mc}\vec{A}\psi^*\psi$ is derived from the Schrödinger equation.

(b) Show that this probability current leads to the same diamagnetic susceptibility obtained from the free energy [(31.25) in the text], $\chi = -\frac{e^2}{4mc^2V}\sum_{filled-states}\langle x^2 + y^2 \rangle$. To do this, assume an orbital axially symmetric along *z* (for example an *s* orbit, with spherical symmetry), and for the vector potential *A* use the gauge choice $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}$, giving a probability current contribution circulating in the *x-y* direction. Note also that an electric current loop has moment $\mu = IS$, where *S* is the area and *I* the electric current. Start with the moment of a thin ring circling *z* with cross section *dS*, and integrate to find the induced moment of the orbital. Note: the area integration over *r* and θ in the current summation can easily be converted to a corresponding volume integration, since the φ integral gives 2π due to axial symmetry—in this way the result can be made to be equivalent to the expectation value. Show that this leads to the susceptibility expression given above.

(c) Show how equation (34.29), $\vec{J}_{el} = -\left[\frac{2e^2}{mc}\vec{A} + \frac{e\hbar}{m}\vec{\nabla}\phi\right]|\psi|^2$, in the Ginzburg-Landau model, results from the probability-current expression above. In this case, assume the wavefunction is $\psi = |\psi| e^{i\phi}$, where $|\psi|$ has constant magnitude so that all changes in ψ come about through the phase, ϕ. Since for superconductors $|\psi|^2$ represents the density of pairs rather than single particles, there are two substitutions needed in order to obtain the desired current. What are these subsitutions?

3) Ashcroft & Mermin problem 34.1