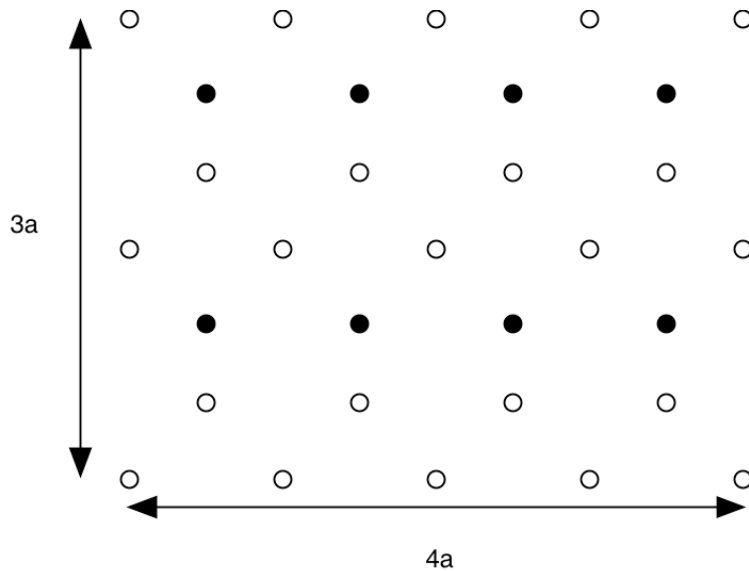


(1) The figure shows a two-dimensional periodic lattice, containing A atoms (white) and B atoms (black). The section of lattice shown makes a $3a \times 4a$ rectangle, as shown (measured from A atom to A atom).

- (a) Indicate a primitive cell.
- (b) Draw clearly a set of primitive lattice vectors
- (c) How many atoms of each type form the basis set?
- (d) Sketch underneath this figure a reciprocal lattice corresponding to this lattice, and indicate two primitive lattice vectors for the primitive lattice (make these reciprocal to your direct lattice vectors). What are the dimensions of the two primitive vectors?



- (2) A two-dimensional solid has an electron energy band of the form,

$$\varepsilon(k) = \frac{\hbar^2}{2} \left(\frac{k_x^2}{m_1} + \frac{k_y^2}{m_2} \right). \quad [1]$$

The masses m_1 and m_2 are constants.

- (a) Derive the semiclassical real-space velocity $\vec{v}(k)$, for electrons in this metal.
- (b) In terms of a general energy ε , what is the number of electron states per volume inside the constant-energy perimeter in k -space? (Note, ellipse area is given on front cover.)
- (c) Using your result from above, find the density of states for this band.
- (d) Determine the Fermi energy, if there is one electron per cell, with unit cell size $a \times b$.

- (3) X-ray scattering in a simple cubic lattice:
- (a) What is the longest x-ray wavelength that will constructively scatter from a simple cubic lattice of unit cell side a ?
 - (b) If the wavelength actually used is slightly shorter than $(1/\sqrt{3})$ times the value from (a), how many scattering lines become visible, in an x-ray powder experiment? What families of lattice planes are responsible for these lines, and what are the scattering angles?
 - (c) Same question as (b), with same wavelength, but for an FCC lattice of cubic cell side a .

(4) For a one-dimensional crystal with two alternating atom types, masses M_1 and M_2 , the vibrational modes are worked out on page 311 of the text. For such a case, in terms of M_1 , M_2 and the spring constant K :

(a) Find the speed of sound.

(b) Suppose a photon beam impinges on the crystal, moving along the chain direction. If the atom masses and spring constant are equivalent to those of typical solids, at what nonzero frequency can the photon frequency equal a phonon frequency, if the photon and phonon wavevectors (k) are also the same.

(c) Find the vibrational density of modes, in the low-frequency limit.

(d) Using your result from (c), find the average energy contained in these vibrational modes at low temperatures (e.g. not precisely $T = 0$, rather the leading non-zero T -dependent energy). You can leave the integral un-evaluated, but factor out the temperature so that the T -dependence is shown explicitly.

(5) Consider a rectangular 2D lattice with lattice vectors a (in the x -direction) and b (in the y -direction), having the specific ratio $b = \sqrt{2}a$.

(a) Plot the free-electron band dispersion curves along the direction $(1, 0)$, in the reduced-zone scheme. Do this up to an energy $5 \frac{\hbar^2}{2m} \left(\frac{\pi}{a}\right)^2$. (Be careful to fold in the bands from all directions.)

(b) In the nearly-free electron model, write expressions for the energies for all of the solutions you found at the point $k = 0$ in part (a), to first order in U , where U is the crystal lattice potential. Write your expression in terms of m , a , and the Fourier coefficients of U . Which Fourier coefficients are involved in each case?

(6) A semiconductor contains both donors and acceptors, with densities N_d and N_a . Assume that in this case $N_d = 3N_a$. The valence and conduction band edges are the usual ϵ_v and ϵ_c , with $\epsilon_g = \epsilon_c - \epsilon_v$ the energy gap, and ϵ_d and ϵ_a the donor and acceptor energies respectively.

- (a) At $T = 0$, what must be the electron density (n) in the conduction band, and the fraction of ionized donors (f_d)?
- (b) What is the position of the chemical potential at $T = 0$?
- (c) As the temperature is raised from $T = 0$, as long as ϵ_d and ϵ_a are relatively close to the respective band edges, there will be a plateau in carrier density vs. T , where the density of carriers in the conduction band becomes nearly T -independent. What will be the density of carriers on this plateau?
- (d) Write an expression for the density of holes in the valence band, for temperatures on the plateau described in (c).