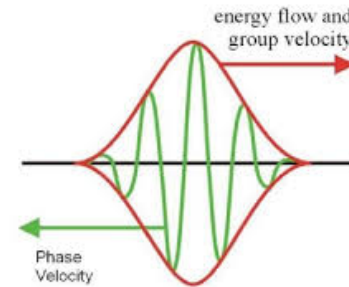


# Semiclassical Electron Dynamics:

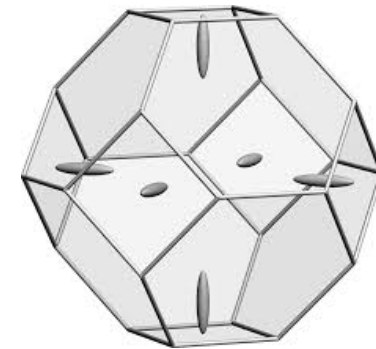
## ○ Group velocity

$$\vec{v}_g = \frac{1}{\hbar} \vec{\nabla}_k \varepsilon$$



## ○ Effective mass:

$$[M^{-1}]_{\alpha\beta} = \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon}{\partial k_\alpha \partial k_\beta}$$



Silicon conduction-band pockets (M. Marder)

## ○ Lorentz force equation:

$$\hbar \dot{\vec{k}} = -e \left[ \vec{E} + \frac{1}{\hbar c} \vec{\nabla}_k \varepsilon \times \vec{B} \right]$$

Progression to adjacent states in same band.

- correct for sufficiently small fields.
- electric transport properties; also Bloch oscillations (perfect conductor); magnetic quantization effects.

# Semiclassical Electron Dynamics

- Lorentz force equation:

$$\hbar \dot{\vec{k}} = -e \left[ \vec{E} + \frac{1}{\hbar c} \vec{\nabla}_k \varepsilon \times \vec{B} \right]$$

- Requires fields sufficiently small for wavepacket to remain within single band.
- $E$  fields: normally difficult to exceed this criterion (Zener breakdown);  $B$  fields: “magnetic breakdown” sometimes important (several-T fields)
- One consequence: Bloch Oscillations; “no electrical conductivity in perfect crystal”

# Bloch oscillations & Transport

- “Paradox”: scattering required for charge transport.
- Bloch oscillations seen only in special cases (superlattices, also cooled-atom lattices); need period  $< \tau$ .

$$\hbar \dot{\vec{k}} = -e\vec{E} \quad \vec{v}_g(\vec{k}) = \frac{1}{\hbar} \vec{\nabla}_k \varepsilon(\vec{k}) \Rightarrow \vec{r}(t) = \int \vec{v}(\vec{k}(t)) dt$$

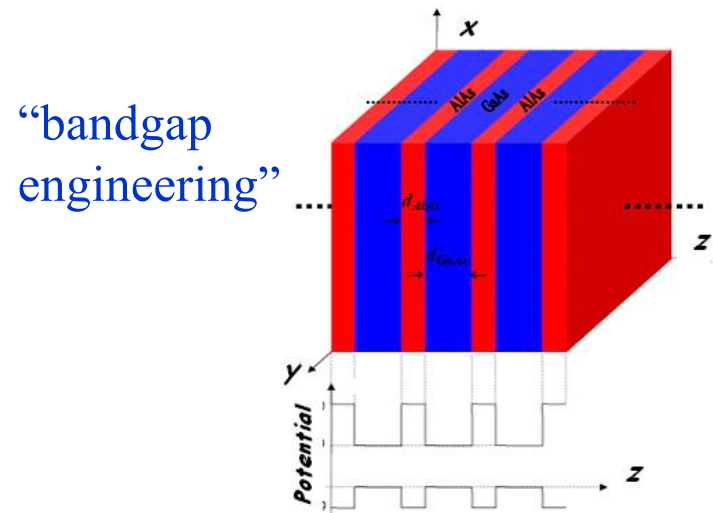
- Zener tunneling: beyond semi-classical, carriers jump bands. (Extreme quantized: Wannier-Stark ladder in strong  $E$  field.)

Magnetic cases: Magnetic breakdown;

Shubnikov-de Haas oscillations.

- Esaki idea: superlattice oscillator

Based on Bloch oscillations



## Transport in metals:

**Electrical conductivity:** typically small “displacement” of FS showed before:

$$k_F = \sqrt[3]{3\pi^2 n} \Rightarrow \begin{aligned} \varepsilon_F &= \hbar^2 k_F^2 / 2m = \hbar^2 (3\pi^2 n)^{2/3} / 2m \\ g(\varepsilon_F) &= \frac{\sqrt{2}}{\pi^2} \left( \frac{m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon_F} = \frac{m(3\pi^2 n)^{1/3}}{\hbar^2 \pi^2} \end{aligned}$$

Result for spherical FS :

$$\sigma = \frac{ne^2\tau}{m} = \frac{2}{3} g(\varepsilon_F) \varepsilon_F \frac{e^2\tau}{m} = \frac{g(\varepsilon_F) v_F^2 e^2 \tau}{3}$$

Effective mass

Ellipsoidal pocket:  
replace by  
 $(m_1 m_2 m_3)^{1/2}$

# Holes and “Hole bands”

- Hole band as “inverted band structure”.

$$\begin{array}{ll} \vec{k} \rightarrow -\vec{k} & \varepsilon \rightarrow -\varepsilon \\ -e \rightarrow +e & m^* \rightarrow -m^* \end{array} \quad \hbar \dot{\vec{k}} = e \left[ \vec{E} + \frac{1}{\hbar c} \vec{\nabla}_k \varepsilon \times \vec{B} \right]$$

- Response of positive-charge holes equivalent to response of all the remaining filled electron states; Lorentz force and mass are *reversed*.
- Normally useful only for small pockets, e.g. nearly-full bands.

